Trajectory Tracking of Robotic Manipulators Using Optimal Sliding mode control

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Abstract—**This paper is concerned with the Optimal Sliding Mode Control (OSMC) for trajectory tracking of Multiple-Input Multiple Output (MIMO) systems. Integrating the first-order sliding mode control (SMC) with Linear-Quadratic-Regulator (LQR) is employed to design an optimal sliding manifold and a global robust control law for asymptotically stabilizing the system to a desired trajectory. The applicability and the efficiency of our approach are illustrated by simulation results for 2-link robot manipulators to show its effectiveness.**

Keywords—sliding mode control; robotic manipulators; LQR; integral sliding surface; tracking trajectory; optimal control; optimal sliding mode control..

I. Introduction

Trajectory tracking control for (MIMO) nonlinear systems has attracted a great deal of attention during the past decades [1, 2]. Compared with single-input single-output (SISO) systems, the tracking control for MIMO nonlinear systems is much more crucial because the output variables are more than one and usually coupled. Robot manipulators, that being usually used in industrial task, are well known by nonlinear models with disturbances and uncertainties. Such systems perform repetitive tasks in many manufacturing applications. For that, the design of ideal controllers for these systems is a challenge for control engineers. Many researches have been followed in order to cope with this problem, such as, for instance, feedback linearization [3], model predictive control [4], and sliding mode control [5].

SMC is known with its robustness against modeled dynamics, nonlinearities and disturbances. As well, it has many good performances such as fast response, good transient performance, and easy realization. The design procedure of a SMC system is to choose a set of switching and to determine a switching control law that forces the system's trajectories onto the sliding surface and maintains the trajectories on it. However, before the sliding surface is reached, the system does not possess the property of being insensitive to noise and uncertainty [6, 7]. As a possible solution to this problem, a new integral sliding mode control [8] (ISMC) to output tracking nonlinear problems with disturbances compensation and uncertainties rejection is proposed.

The contribution of this paper is twofold. At first, the tracking problem of nonlinear system, such as a robot, is transferred into an optimal state regulation problem about linear error system. For that, an error equation is constructed. Second, we consider the problem of designing a robust control law to achieve optimal performance and reject the disturbances for the robot. The LQR approach [9] incorporated with the first order-sliding mode control is considered. Based on optimal control law, an optimal sliding manifold is constructed, which can remove the reaching phase of conventional SMC effectively and guarantee the global sliding mode.

The paper is organized as follows:
The robot dynamics is presented in Section 2. The main results on designing an optimal sliding surface and a robust control law are presented in Section 3. Results on robot tests are reported in Section 4 and the paper ends with the concluding remarks in Section 5.

II. Robot Dynamics

The dynamic model of a 2- link rigid robot, given from the Lagrange equation system, is described by the following matrix equation [10, 12]:

$$B(q)\ddot{q}+C(q,\dot{q})\dot{q}+G(q)=\tau \tag{1}$$

where $q=[q_1 \quad q_2]\in\Re^2$ is the joint position vector, $\dot{q}\in\Re^2$ is the joint velocity vector, $\ddot{q}\in\Re^2$ is the joint acceleration vector, τ is the input torque, $B(q)\in\Re^{2\times2}$ is a positive definite inertia matrix, $C(q,\dot{q})$ is the Coriolis and centrifugal loading vector, and $G(q)$ is the gravitational loading vector.
The matrix $B(q)$ is defined as follow:

$$B(q)=\begin{bmatrix} b_{11}(q_2) & b_{12}(q_2) \\ b_{21}(q_2) & b_{22} \end{bmatrix} \tag{2}$$

The matrix $C(q,\dot{q}), G(q)$ are respectively defined as:

$$C(q,\dot{q}) = \begin{bmatrix} c_{11}(q_2,\dot{q}_2) & c_{12}(q_2,\dot{q}_2) \\ 0 & c_{22}(q_2,\dot{q}_2) \end{bmatrix} \quad (3)$$

$$G(q) = \begin{bmatrix} G_1(q_1,q_2) & G_2(q_1,q_2) \end{bmatrix}^T \quad (4)$$

where:

$$\begin{aligned}
b_{11}(q_2) &= (m_1+m_2)l_1^2 + m_2 l_2^2 + 2m_2 l_1 l_2 \cos(q_2) + J_1 \\
b_{12}(q_2) &= m_2 l_2^2 + m_2 l_1 l_2 \cos(q_2) \\
b_{21}(q_2) &= m_2 l_2^2 + m_2 l_1 \cos(q_2) \\
b_{22} &= m_2 l_2^2 + J_2 \\
c_{11}(q_2) &= -2m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\
c_{12}(q_2) &= -m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\
c_{21} &= 0 \\
c_{22}(q_2) &= m_2 l_1 l_2 \sin(q_2)\dot{q}_2 \\
G_1(q_1,q_2) &= (m_1+m_2) l_1 g \cos(q_1) + m_2 l_2 \cos(q_1+q_2) \\
G_2(q_1,q_2) &= m_2 l_2 \cos(q_1+q_2)
\end{aligned} \quad (5)$$

where l_k, $k=1,2$ is the distance from joint to the center of mass of link k, $m_k, k=1,2$ is the mass of link k, $J_k, k=1,2$ is the moment of inertia of link k and g is the gravitational constant.

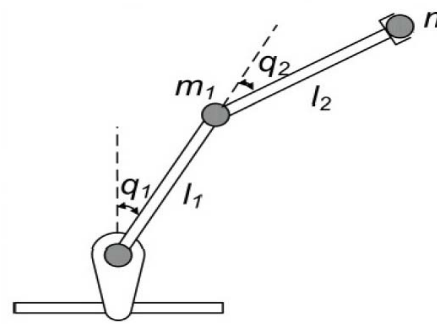

Fig. 1. Configuration of a two-link robotic manipulator.

From (1), it is easy to show that:

$$\ddot{q} = [B(q)]^{-1}[\tau - C(q,\dot{q}) - G(q)] \quad (6)$$

Assume that there exists τ_d which satisfies [11]:

$$\ddot{q}_d = [B(q_d)]^{-1}[\tau_d - C(q_d,\dot{q}_d) - G(q_d)] \quad (7)$$

where q_d is the desired trajectory and τ_d can be calculated from (7) by using the desired trajectory as:

$$\tau_d = B(q_d)\ddot{q}_d + C(q_d,\dot{q}_d)\dot{q}_d + G(q_d) \quad (8)$$

Subtracting (7) from (6) yields:

$$\begin{aligned}
\ddot{q}_d - \ddot{q} &= [B(q_d)]^{-1}\tau_d + [B(q)]^{-1}C(q,\dot{q})\dot{q} + [B(q)]^{-1}G(q) \\
&\quad - [B(q_d)]^{-1}C(q_d,\dot{q}_d)\dot{q}_d - [B(q_d)]^{-1}G(q_d) - [B(q)]^{-1}\tau
\end{aligned} \quad (9)$$

therefore:

$$\ddot{q}_d - \ddot{q} = [B(q_d)]^{-1}(\tau_d - \tau) - [B(q_d)]^{-1}C(q_d,\dot{q}_d)(\dot{q}_d - \dot{q}) + [B(q_d)]^{-1}\rho \quad (10)$$

where ρ is expressed as:

$$\begin{aligned}
\rho &= ([B(q_d)][B(q)]^{-1}C(q,\dot{q}) - C(q_d,\dot{q}_d))\dot{q} + [B(q_d)][B(q)]^{-1}G(q) \\
&\quad - ([B(q_d)][B(q)]^{-1} - I)\tau - G(q_d)
\end{aligned} \quad (11)$$

with I is the identify matrix.

Introducing the state vector x:

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} (q_d - q)^T & (\dot{q}_d - \dot{q})^T \end{bmatrix}^T \quad (12)$$

The system of differential equation (10) can be written as follow:

$$\dot{x} = G(q_d,\dot{q}_d)x + H(q_d)u + H(q_d)\rho \quad (13)$$

where x is the state vector, u the new input vector stabilizing the system (13), $G(q_d,\dot{q}_d)$ is the state matrix, and $H(q_d)$ is a gain matrix with:

$$G(q_d,\dot{q}_d) = \begin{bmatrix} 0 & I \\ 0 & -[B(q_d)]^{-1}C(\dot{q}_d,q_d) \end{bmatrix} \quad (14)$$

$$H(q_d) = \begin{bmatrix} 0 \quad 0 \\ 0 \quad 0 \\ [B(q_d)]^{-1} \end{bmatrix} \quad (15)$$

The new control law u can be represented as follow:

$$u = \tau_d - \tau \quad (16)$$

The discretization of the model (13) with a sampling time gives:

$$x(k+1) = G_d x(k) + H_d u(k) + H_d \rho(k) \quad (17)$$

where:

$$G_d = I + T_e G(T_e k) \quad (18)$$

$$H_d = T_e H(T_e k) \quad (19)$$

T_e is the sampling time.

III. Optimal sliding mode control

A. Classical sliding mode control (CSMC)

The classical sliding function [12] vector is given by the following expression:

$$\sigma(k) = Dx(k) \quad (20)$$

with $D \in \Re^{m \times n}$.

We define the control law as:

$$u(k) = u_{eq}(k) + u_n(k) \quad (21)$$

The equivalent control law, that forces the system to reach the sliding surface is obtained by equating $\sigma(k+1) = 0$ to zero, is given by:

$$u_{eq}(k) = -(DH_d)^{-1}(DG_d x(k) + DH_d \rho(k)) \quad (22)$$

where (DH_d) is invertible.

The robustness is ensured by the addition of a discontinuous term expressed as follow:

$$u_n(k) = -R sign(s(k)) \quad (23)$$

where: $sign(s(k)) = [sign(s_1) \ \ldots \ sign(s_m)]^T$ and R is a diagonal positive matrix.

The classical control law is:

$$u(k) = -(DH_d)^{-1}(DG_d x(k) + DH_d \rho(k)) - R sign(s(k)) \quad (24)$$

The most inconvenient of this control law, that during the reaching phase, the system may be sensitive to these disturbances. For that, to achieve optimal performance and reject these disturbances for nonlinear systems, an optimal sliding surface is proposed in the next section.

B. Design of robust optimal sliding surface

Considering the system (17), we choose the integral discrete sliding surface as follows:

$$\sigma(k) = Dx(k) - Dx(0) + h(k) \quad (25)$$

where $D \in \Re^{m\times n}, h(k) \in \Re^m$ and $x(0)$ is the initial condition of the system.
$h(k)$ is calculated as:

$$h(k) = h(k-1) - D(G_d - H_d E - I)x(k-1) \quad (26)$$

with: I is the identify matrix.
The matrix E is to be chosen later.
Let $u_0(k) = -Ex(k)$ and u_0 can minimize the following performance index:

$$J = \frac{1}{2}\sum_{k=0}^{N-1} x^T(k)Qx(k) + u_0^T(k)Ru_0(k) \quad (27)$$

where $Q \in \Re^{n\times n}$ is a symmetric positive definite matrix, $R \in \Re^{m\times m}$ is a positive definite matrix of designer's choice.

To find the optimal feedback controller $u_0(k)$ the Hamiltonian is designed as follow [9]:

$$H(k) = \frac{1}{2}(x^T(k)Qx(k) + u^T(k)Ru(k)) + \lambda(k+1)(G_d x(k) + H_d u_0(k)) \quad (28)$$

For the optimality condition of the discrete-time system, we have three conditions:

- **State equation :**

$$x(k+1) = \frac{\delta H(k)}{\delta \lambda(k+1)} = (G_d x(k) + H_d u_0(k)) \quad (29)$$

- **Costate equation :**

$$\lambda(k) = \frac{\delta H(k)}{\delta x(k)} \quad (30)$$

- **Stationary condition :**

$$\frac{\delta H(k)}{\delta u_0(k)} = 0 = Ru_0(k) + H_d^T \lambda(k+1) \quad (31)$$

In order to find the optimal controller to handle the states and desired trajectories, it is reasonable to assume that the costate can be expressed by:

$$\lambda(k) = P(k)x(k) \quad (32)$$

Then, the optimal controller is derived from the stationary condition:

$$\begin{aligned} u_0(k) &= -R^{-1}H_d^T \lambda_{k+1} = -R^{-1}H_d^T P(k+1)x(k+1) \\ &= -R^{-1}H_d^T P(k+1)(G_d x(k) + H_d u_0(k)) \\ &= -(R + H_d^T P(k+1)H_d)^{-1} H_d^T P(k+1)G_d x(k) \\ &= -Ex(k) \end{aligned} \quad (33)$$

The matrix E is expressed as follow:

$$E = (R + H_d^T P(k+1)H_d)^{-1} H_d^T P(k+1)G_d \quad (34)$$

P is the solution of the discrete algebraic Riccati equation:

$$P = Q + G_d^T P G_d - G_d^T P H_d (H_d^T P H_d + R)^{-1} H_d^T P G_d \quad (35)$$

The dynamics of the closed-loop system for nominal system without disturbances is:

$$\begin{aligned} x(k+1) &= G_d x(k) + H_d u_0(k) \\ &= (G_d - H_d E)x(k) \end{aligned} \quad (36)$$

From (25), the following equation is obtained:

$$\begin{aligned} \sigma(k+1) &= Dx(k+1) - Dx(0) + h(k+1) \\ &= D(G_d x(k) + H_d u(k) + H_d \rho(k)) - Dx(0) + h(k) \\ &\quad - D(G_d - H_d E - I)x(k) \\ &= DH_d u(k) + DH_d \rho(k) - Dx(0) + h(k) + D(H_d E + I)x(k) \end{aligned} \quad (37)$$

The equivalent control $u_{eq}(k)$ that provides $\sigma(k+1) = 0$ can be found from (37) as:

$$u_{eq}(k) = -(DH_d)^{-1}\left[h(k) + D(H_d E + I)x(k) + DH_d \rho(k) - Dx(0)\right] \quad (38)$$

Obviously, such equivalent control is not feasible since it requires exact knowledge of disturbance $\rho(k)$, which is generally unavailable in practice. Instead, $\rho(k)$ can be estimated by its previous value $\rho(k-1)$. Let:

$$H_d \rho(k) \approx H_d \rho(k-1) = x(k) - G_d x(k-1) - H_d u(k-1) \quad (39)$$

Substituting (38) into (17), the sliding mode dynamics becomes:

$$x(k+1) = G_d x(k) + H_d u(k) + H_d \rho(k)$$
$$= H_d\left[-(DH_d)^{-1}((h(k) + D(H_d E + I)x(k) + DH_d\rho(k) - Dx(0))\right] + G_d x(k) + H_d\rho(k)$$
$$= G_d x(k) - H_d (DH_d)^{-1} h(k) - H_d (DH_d)^{-1}(DH_d)Ex(k) - H_d (DH_d)^{-1} Dx(k) - H_d\rho(k) + H_d (DH_d)^{-1} Dx(0) + H_d\rho(k)$$
$$= G_d x(k) - H_d Ex(k) - H_d (DH_d)^{-1}\left[h(k) - Dx(0) + Dx(k)\right]$$
$$= G_d x(k) - H_d Ex(k) - H_d (DH_d)^{-1}\sigma(k) \quad (40)$$

In sliding mode we have $\sigma(k+1) = \sigma(k) = 0$ .Considering (40), we can obtain:

$$x(k+1) = (G_d - H_d E)x(k) \quad (41)$$

Comparing (41) with (36) the sliding mode dynamics of the system (17) and the optimal dynamics of nonlinear nominal system have the same equation. As a result, sliding motion guarantees that the system has complete robustness to these disturbances.

C. Design of robust control law

To ensure the robustness of the system (17) we propose the control law as follow:

$$u(k) = u_0(k) + u_n(k) = -Ex(k) - Rsign(s(k)) \quad (42)$$

The nominal control u_0 takes care of the nominal plant dynamics; it is used to achieve good transient performance. u_n is the discontinuous part, which provides complete compensation for disturbances of system (17).

IV. Simulation results

In order to verify the performance of the proposed control strategy based on the (ISMC) approach, some simulations have been developed using MATLAB. The proposed control law is applied for trajectory tracking of a two-link robotic manipulator defined at the section 2. The reference signals are:

$$q_{d1}(t) = 1.25 - \frac{7}{5}e^{-t} + \frac{7}{20}e^{-4t}, q_{d2}(t) = 1.25 - e^{-t} + \frac{1}{4}e^{-4t}$$

Therefore, the 2-link robotic manipulator has four inner states:

$$x = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} (q_d - q)^T & (\dot{q}_d - \dot{q})^T \end{bmatrix}^T$$

The system is sampled with $T_e = 0.05s$.

The control is required to optimize the quadratic performance index with $Q = diag(60,60,60,60)$ and $R = diag(1,0.5)$.

"Fig. 2-9" provide a comparison of the system responses using classical sliding mode control (CSMC) and the proposed optimal sliding mode control (OSMC) technique.

The response of the states x_1, x_1, x_3, x_4 are plotted in figures *2, 3, 4, 5* that confirms that these converge to zero quickly.

Fig. 6-7 show the evolution of the output q_1 and q_2 . We can notice that both output follows their desired reference.

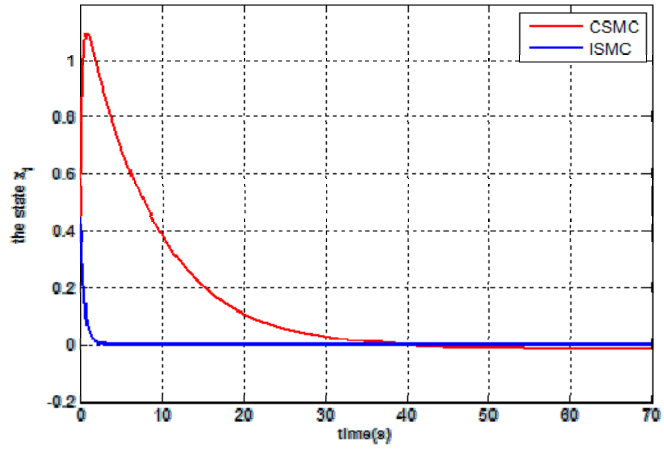

Fig. 2. The tracking error x_1 .

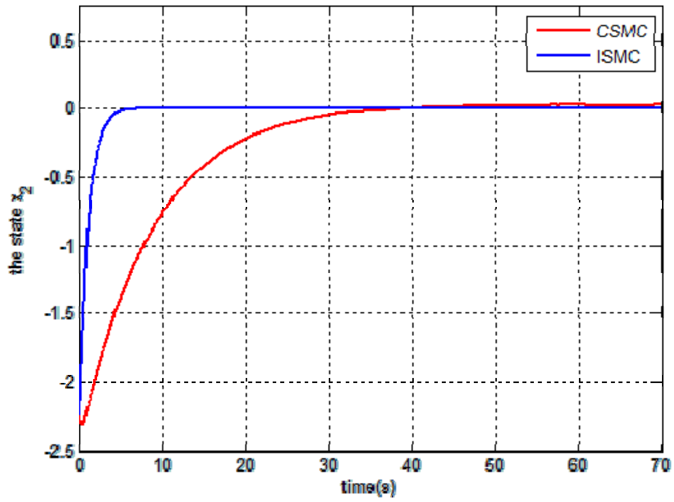

Fig. 3. The tracking error x_2 .

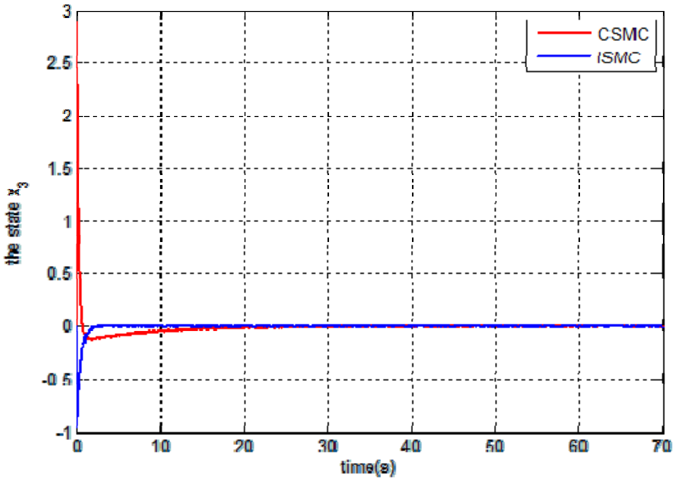

Fig. 4. The tracking error x_3

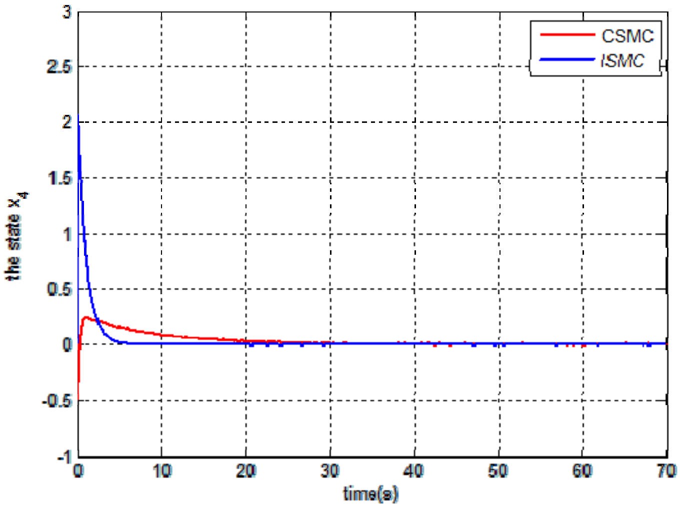

Fig. 5. The tracking error x_4 .

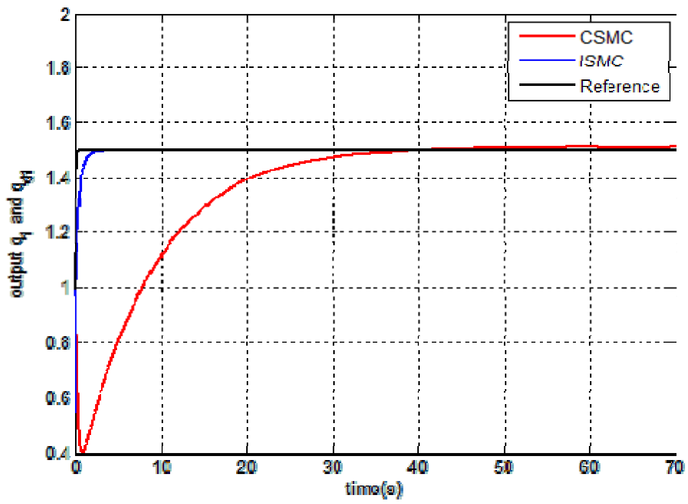

Fig. 6. Reference trajectory and trajectory followed by the joint position q_1 .

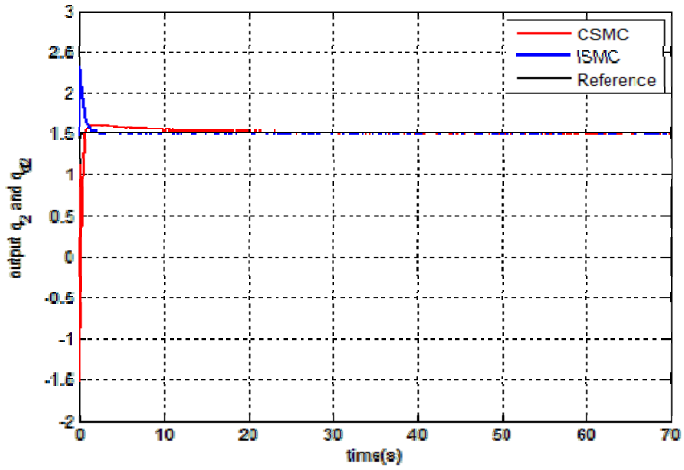

Fig. 7. Reference trajectory and trajectory followed by the joint position q_2 .

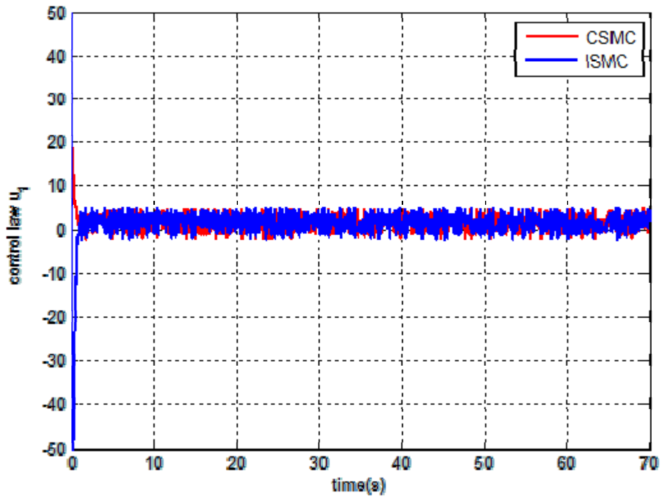

Fig .8. Evolution of the control law u_1 .

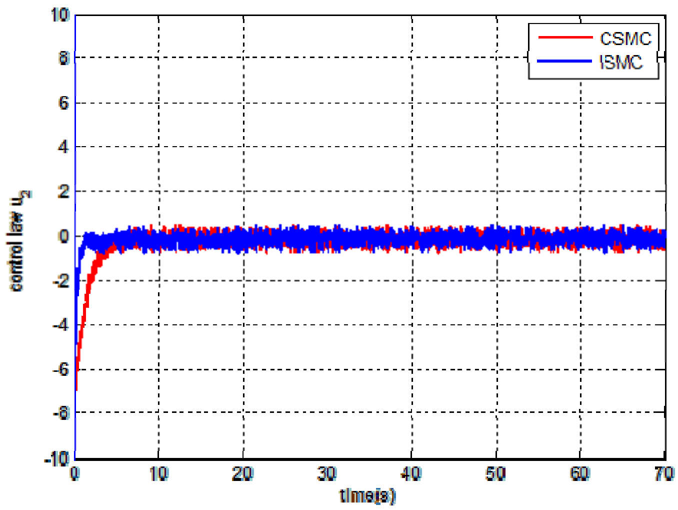

Fig .9. Evolution of the control law u_2 .

It can be seen that the controlled system could track the desired trajectory by both controllers, thus, the OSMC provides better features than CSMC in terms of robustness to system disturbances.

V. Conclusion

A robust optimal tracking control for robotic manipulator has been studied, a linear tracking error has been established and the nonlinear optimal tracking problem was transformed into a linear problem. Therefore, a control law, based on the ISMC and LQR approach, has played an important role to improve the performances to compensate the disturbances and to ensure the robustness of the system simulations.

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