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Dynamic Allocation of Backhaul Resources in Converged Wireless-Optical Networks

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***Abstract*—The market uptake of 4th Generation Networks is expected to support the increasing demand for wireless broadband services and ensure an enhanced mobile user experience. In this direction, the convergence of a wireless access network with an optical backhauling has been proposed. However, in such a converged architecture, the traditional fixed commitment of the backhaul resources does not prove to be as efficient, and novel dynamic schemes are required that consider both the needs of the base stations and the limitations of the passive optical network. The present paper is concerned with the topic of resource allocation in two competing base stations that belong to different operators and share a common optical backhaul network infrastructure. An approach based on evolutionary game theory is proposed and employed, with a view to examining the interactions among the base stations and the passive optical network. Using the model of replicator dynamics, the proposed system design is proven to be asymptotically stable. In addition, the paper studies and reveals the extent to which time delay can have an impact on the proposed system design.**

***Index Terms*—Resources allocation, network convergence, multiple operators, game theory, time delay, optical backhaul.**

I. Introduction

In recent years, mobile operators (MOs) are confronted with a continuous growth of new bandwidth consuming applications, resulting in increased needs in terms of provisioned data rates in order to ensure a high quality of experience to mobile users [1]. The market uptake of 4th Generation (4G) Networks and the upcoming 5G networks offer the possibility to satisfy this demand, typically providing increased capacity, low latency and seamless mobility [2], [3]. In this direction, the convergence of the wireless access network with a wired optical network (xPON) in the backhaul has been proposed [4], [5]. Assuming that backhaul resources are properly managed and allocated in an efficient manner, the high throughput typically provided by the optical backhaul network can contribute into the accomplishment of the 4G vision for offering high Quality of Service (QoS) to mobile users [6].

The aforementioned appropriate allocation of backhaul network resources becomes a major issue of concern for the MOs in the context of enhanced QoS provisioning [7]. Traditionally in telecommunications engineering, network planning has been static, leading to a flat and fixed scheme for resources commitment, which -despite its simplicity- has yielded quite decent results in previous mobile network generations. However, such methods are not as efficient within a converged optical/wireless network, considering also that the backhaul optical network might belong to a different operator or that many customers might share its use. Hence, a more effective approach would involve the dynamic calculation and assignment of necessary resources to each base station (BS). The motivation for such a dynamic approach also stems from the self organisation paradigm [8] in mobile networks, which gains momentum and requires from BSs to be smarter and equipped with non-negligible processing capabilities, as opposed to the traditional centralised schemes in which a limited set of central authorities is responsible for a large number of functions in the entire network. This notion of smart BSs that sense their environment, using properly reasoning and adapting their behaviour, paves the way for increased robustness and efficiency.

There is currently a number of scientific studies dealing with the issues of efficient resource allocation at the backhaul network part. In [9], the authors investigate the problem of fairness in backhaul resource allocation, focusing on the out-band relay case in LTE Advanced Networks, studying and comparing the performance of different resource allocation strategies with regards to throughput fairness between backhaul and macro access link. In [10], the authors are again concerned with an LTE Advanced system, proposing a buffer level based resource partitioning scheme for the backhaul, which aims at achieving an optimised allocation of resources among relay nodes and macro user equipments. In [11], the author proposes a resource allocation management system that maximises the traffic which can be served by a mobile backhaul network, by using a path optimisation algorithm which consists of two components; an offline component that is based on the predicted traffic demand and an online component for the excess traffic. A game theoretic cooperative scheme for resource allocation is studied in [12], where the authors propose a bankruptcy game based approach for sharing the limited wireless resources among virtual mobile operators. In [13], the authors study the problem of resource allocation in a wireless infrastructure, in which the access part and the backhaul part share the same millimeter wave bands. They investigate several resource allocation algorithms for achieving fairness among stations and maximising the overall network capacity, and they conclude that the system capacity can be improved up to 35% compared to the fixed resource allocation depending on the traffic demand. In [14], the authors study the problem of joint backhaul and access link optimisation for time-division duplexing mode of operation

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and they propose an algorithm for achieving rate balancing based on cell grouping. Though all the above schemes present interesting approaches for the backhaul resource allocation problem, they cannot be directly implemented in the context of converged networks where competing MOs share an optical backhaul infrastructure. Moreover, the existence of service level agreements (SLAs) between the MOs and the PON operator bind the latter regarding the pricing of the resources, thus constituting the auction based approaches inapplicable.

The problem of resource allocation in converged networks is studied in [15]-[19]. In [15], the authors introduce and describe an artificial neural network based approach that enables the prediction of forthcoming traffic demand at the BS side, and proactively requests the needed resources from the optical backhaul network. In [16], the problem of backhaul resource allocation for converged optical/wireless networks is modelled as a game between the subscribers and the BS considering, however, that the PON operator can always satisfy the BS's requests. Another approach inspired by game theory is presented in [17], where the authors investigate the fair allocation of bandwidth in Fibre-Wireless access networks. The authors place more emphasis on the wireless interface and propose an algorithm based on the cooperation of relay nodes in the access network, including gateway and wireless routers. In particular, the main concept requires from the routers to optimise both local traffic (that originates from the router) and foreign traffic (that the router forwards), after having agreed on a set of specific sharing and trusting levels among the nodes. The problem of joint wireless and optical resource allocation in survivable fiber-wireless access networks is studied in [18]. The authors propose a resource allocation approach for providing the guaranteed service under component failure. Initially, they estimate the connection availability and then, they use a joint optical and wireless resource allocation scheme that minimises the occupied resources under the requirement of satisfying the agreed bandwidth of the service. A further approach for resource allocation in converged infrastructures is described in [19]. In particular, the authors study the problem of resource allocation in a wireless optical infrastructure consisting of a Gigabit PON (GPON) and a Wi-Fi access network. Their approach uses time devision multiple access at the optical part and carrier sense multiple access with collision avoidance at the wireless part in order to provide high capacity and support mobility requirements. However, the above approaches are lacking any consideration regarding the limitations at the side of the PON. The resources of the PON are shared among many clients (e.g. business users or other mobile operators) and, thus, the PON operator cannot always satisfy their requested demand, leading to a suboptimal performance for the above schemes.

This paper studies the game theoretic approach of the backhaul resource allocation problem where two competing BSs are served by a common backhaul network infrastructure[1] consisting of a TDMA-PON. In essence, each BS, which belongs to a different MO, estimates the necessary resources and proactively requests their commitment from the PON operator, exploiting the dynamic bandwidth allocation mechanisms of the TDMA-PON [21], in order to provide sufficient QoS to its subscribers. If the aggregated demand of the BSs is below the maximum capacity of the PON, then their requests can be satisfied by the PON operator. On the other hand, if the aggregated demand of the BSs exceeds the capacity of the PON, then their requests cannot be satisfied simultaneously. For this purpose, this paper introduces and discusses on a novel scheme for modelling the dynamic behaviour of BSs, towards accomplishing a suitable allocation of the backhaul resources considering the capacity limitations of the PON. In contrast to the traditional static resource allocation of the backhaul network, next generation wireless-optical converged networks should exploit the dynamic bandwidth allocation capabilities of the PON in order to efficiently manage the resources based on their current needs. Evolutionary game theory [22], [23] is employed for the purpose of studying the interactions among the BSs and the PON. Specifically, the evolutionary game theory is used to describe the growth of the players' strategies. The players are considered as populations of agents using pure strategies, and thus, the probability of a player to use a specific strategy is the proportion of its agents that play this strategy. Contrary to the traditional game theory which provides a static solution that refers to the Nash Equilibria, evolutionary game theory provides a dynamic approach that can be used to show which of these equilibrium points can be eventually achieved. In addition, the proposed model design is examined and proven to be stable under both replicator dynamics as well as delayed replicator dynamics. To the authors' best knowledge, this evolutionary game theoretic approach for the backhaul resource allocation problem considering the capacity limitations of the PON has not yet been addressed in the scientific literature.

The remainder of this paper is structured as follows. Section II presents the model approach of the problem and defines the relevant assumptions. The implementation of the evolutionary game theoretic scheme for the backhaul resource allocation problem is presented in Section III. Section IV examines the impact of time delay to the proposed scheme, whereas extensive and illustrative simulation results are provided in Section V. Finally, Section VI draws and summarises conclusions.

II. Problem Formulation and Relevant Assumptions

Let us consider two BSs belonging to two competing MOs, namely MO_A and MO_B, that share a common backhaul infrastructure [6] consisting of a PON owned by a different operator[2], as depicted in Figure 1. Each BS can estimate the necessary resources' demand and proactively request their commitment from the PON in order to provide satisfactory QoS to its subscribers. The estimation can be done using forecasting methods as explained in [15]. Specifically, at certain periods of time, each BS estimates the proper information

[1] Similar network topologies can be found in [4], [20].

[2] It is noted that the proposed analysis is also valid for more than one BSs per MO, assuming that they are all connected to the same optical network unit (ONU), and each ONU serves only one MO. Accordingly, the BS can serve several wirelessly connected small-cell networks. In such scenarios the demand refers to the aggregated demand of the serving BSs.

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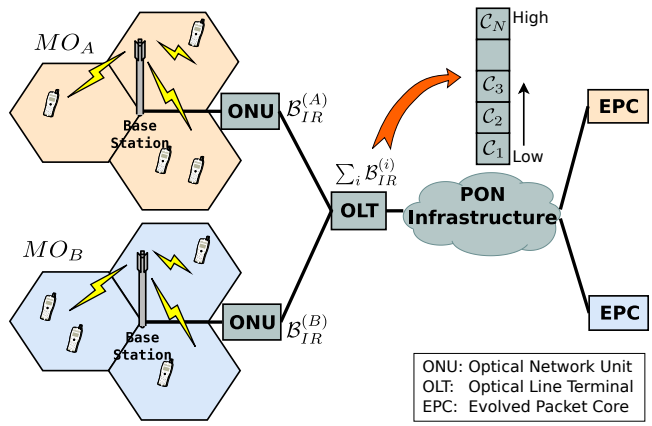

Fig. 1. Network topology.

rate (IR)[3], $\mathcal{B}_{IR}^{(A)}$ and $\mathcal{B}_{IR}^{(B)}$ for MO$_A$ and MO$_B$, respectively, and request its commitment from the PON. The PON receives all the requests, assesses the availability of the resources and commits the necessary IRs if the aggregated demand $\sum_z \mathcal{B}_{IR}^{(z)}$, $z \in \{A, B\}$, is below its maximum capacity $\mathcal{T}$.

Assume for simplicity that the maximum IR that can be served by the PON is divided in N equal chunks. Furthermore, assume that there are discrete serving classes of IR[4] provided by the PON operator, so that $\mathcal{C}_i$ offers an IR which is the i/N ratio of the maximum IR, with $i \in [1, \ldots, N]$. As a result, class $\mathcal{C}_N$ represents the maximum possible IR that can be either offered or requested and class $\mathcal{C}_1$ denotes the lowest IR that can be offered or requested. If the BS of MO$_A$ requests for class $\mathcal{C}_i$, i.e. $\mathcal{B}_{IR}^{(A)} \in \mathcal{C}_i$, and the BS of MO$_B$ requests for class $\mathcal{C}_j$, i.e. $\mathcal{B}_{IR}^{(B)} \in \mathcal{C}_j$, then there are two interesting cases:

- If $\sum_z \mathcal{B}_{IR}^{(z)} \leq \mathcal{T}$, then the aggregated demand of the BSs can be satisfied by the PON operator. It is assumed that, in this case, the MOs experience payoffs $\mathcal{U}_A^{(i)}$ and $\mathcal{U}_B^{(j)}$, respectively, which can mathematically be expressed as

$$\mathcal{U}_A^{(i)} = S_A^{(i)} - C^{(i)} \tag{1}$$

$$\mathcal{U}_B^{(j)} = S_B^{(j)} - C^{(j)} \tag{2}$$

  where $S_A^{(i)}$ and $S_B^{(j)}$ are monotonically increasing functions with regards to the IR, which implies that MOs experience a higher payoff when more resources are secured, due to the higher customer satisfaction and the respective revenues, while $C^{(i)}$ and $C^{(j)}$ indicate the cost of the resources of classes $\mathcal{C}_i$ and $\mathcal{C}_j$, respectively, and depend on the SLAs between the MOs and the PON operator. It is assumed that the pricing policy of the PON operator, concerning the cost for leasing the resources is the same for all the MOs.
- If $\sum_z \mathcal{B}_{IR}^{(z)} > \mathcal{T}$, then the aggregated demand of the BSs cannot be satisfied by the PON operator. Assuming a fair policy by the PON operator, in which no MO is favoured over the other, the PON operator rejects their requests, given that the SLAs are not violated, and a renegotiation phase concerning the allocation of the resources begins. In this case, it is considered that the payoffs experienced by the MOs are zero.

[3]The current work refers to the committed IR (CIR), a formal definition of which can be found in [21], [24], [25].

[4]The granularity of the IRs is also dictated by the backhaul requirements described in [25].

Denoting with $\mathbf{A}$ the payoff matrix of MO$_A$, and with $\mathbf{B}$ the payoff matrix of MO$_B$, based on the analysis above, it holds that

$$\mathbf{A} = \begin{bmatrix} \mathcal{U}_A^{(1)} & \mathcal{U}_A^{(1)} & \cdots & \mathcal{U}_A^{(1)} & 0 \\ \mathcal{U}_A^{(2)} & \mathcal{U}_A^{(2)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathcal{U}_A^{(N-1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

$$\mathbf{B} = \begin{bmatrix} \mathcal{U}_B^{(1)} & \mathcal{U}_B^{(2)} & \cdots & \mathcal{U}_B^{(N-1)} & 0 \\ \mathcal{U}_B^{(1)} & \mathcal{U}_B^{(2)} & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathcal{U}_B^{(1)} & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$$

where $A_{i,j}$ represents the payoff of MO$_A$ when class $\mathcal{C}_i$ is requested by MO$_A$ and class $\mathcal{C}_j$ is requested by MO$_B$. Similarly, $B_{i,j}$ represents the payoff of MO$_B$ when MO$_B$ asks for class $\mathcal{C}_j$ and MO$_A$ for class $\mathcal{C}_i$.

As can be derived from the above analysis, the payoff of the MOs is significantly affected by the cost of the requested resources. Specifically, as the resources requested by the MOs increase, the leasing cost rises. However, the gain $S_z^{(i)}$, $z \in \{A, B\}$, of the MOs for requesting class $\mathcal{C}_i$ depends not only on their contract-based revenues for providing QoS to their customers, but also on their subscribers' satisfaction, which increases as more resources are available for them. As a result, assuming that the MOs give more weight on their subscribers' satisfaction rather than the cost of the leasing resources, the payoffs $\mathcal{U}_A^{(i)}$ and $\mathcal{U}_B^{(i)}$ for requesting class $\mathcal{C}_i$ and $\mathcal{C}_j$, respectively, are monotonically increasing functions of the IR. This assumption corresponds to the overdimensioning of the resources by the MOs in order to provide an enhanced QoS to their end-users. However, in Subsection III-B this assumption loosens and the behaviour of the system is presented.

III. Evolutionary Allocation of Backhaul Resources

A. Evolutionary game dynamics model

In order to apply the EGT approach to the problem of backhaul resource allocation, assume that x_i denotes the frequency of the population of MO$_A$ using class $\mathcal{C}_i$, $i \in [1, \ldots, N]$. Respectively, y_j denotes the frequency of the population of MO$_B$ using class $\mathcal{C}_j$, $j \in [1, \ldots, N]$. It holds that $\sum_i x_i = 1$ and $\sum_j y_j = 1$ and as a result, it can be considered that x_i and y_j depict the probability of pure strategy i and j being used within each population, i.e. the probability of MO$_A$ and MO$_B$ to use class $\mathcal{C}_i$ and $\mathcal{C}_j$, respectively. Furthermore, assume that the MOs reconsider their strategies in certain periods, i.e.

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generations, and only the strategies leading to a payoff higher than average are favoured. In case the periods of time are considered small enough, the rate by which the strategies of the MOs change can properly be expressed by means of the replicator dynamics equations as

$$\dot{x_i} = x_i \cdot \left(\sum_{j=1}^{N} A_{i,j} y_j - \sum_{k=1}^{N} \sum_{m=1}^{N} x_k A_{k,m} y_m \right) \tag{3a}$$

$$\dot{y_j} = y_j \cdot \left(\sum_{i=1}^{N} B_{i,j} x_i - \sum_{k=1}^{N} \sum_{m=1}^{N} y_k B_{m,k} x_m \right) \tag{3b}$$

for $i, j = 1, \ldots, N$, where the first term inside the brackets is the payoff of the pure strategy i and j for the MO_A and MO_B, respectively, while the second term is the corresponding average payoff. The validity of the following theorem can be proven.

Theorem 3.1: The aggregated requests made by the MOs converge to the available resources of the PON.

Proof: The proof is included in Appendix A. ■

This theorem affirms that the designed game will be settled to a rest point corresponding to a pure strategy Nash Equilibrium[5] and will not oscillate chaotically, meaning that the MOs' requests for resources will converge to an equilibrium point without any coordination between them. Hence, the interactions between the MOs and the PON operator will evolve to a system state in which the aggregated demand of the MOs can be satisfied by the PON operator.

B. Overpricing of the resources

In the previous subsection, it was assumed that the payoffs $\mathcal{U}_A^{(i)}$ and $\mathcal{U}_B^{(j)}$ of the MOs for requesting classes $\mathcal{C}_i$ and $\mathcal{C}_j$, respectively, were monotonically increasing functions of the IR. This assumption implied that the increase in the leasing cost of the resources $C^{(i)}$ is less than the increase in the payoff $S_z^{(i)}$, $z \in \{A, B\}$, experienced by the MOs for requesting class $\mathcal{C}_i$. However, this case is not always valid. Specifically, it is expected that the cost for leasing a great portion of the resources can be prohibitively high.

As a result, there exist threshold classes $\mathcal{C}_{thr}^A$ and $\mathcal{C}_{thr}^B$ for the MO_A and the MO_B, respectively, above which the request for resources becomes prohibited due to the high cost. Assuming that the pricing policy of the PON operator does not result in any unexploited resources, as the operator's revenues are maximised when all the resources have been allocated, Theorem 3.2 holds

Theorem 3.2: The MOs request for certain classes of IR $\mathcal{C}_i$ and $\mathcal{C}_j$ that are below their respective thresholds $\mathcal{C}_{thr}^A$ and $\mathcal{C}_{thr}^B$ and the aggregated demand converges to the available resources of the PON.

Proof: The proof is included in Appendix B. ■

Theorem 3.2 is related to the realistic case of the resources being characterised by a leasing cost that is prohibitively high as the requested amount increases. However, it affirms that the game will again be settled to an equilibrium point where the resources will be more equally shared between the MOs.

[5]It is reminded that the Nash Equilibrium describes a state where each player's strategy is optimal given the strategies of all other players [26].

C. Discussion

In Subsection III-A, it was proven that the aggregated demand of the MOs will converge to the available resources under the model of replicator dynamics. In order to highlight and interpret the applicability of this result, let us consider that MO_A requests for resources belonging to class $\mathcal{C}_i$, while MO_B requests for resources belonging to class $\mathcal{C}_j$[6]. If the aggregated demand of the MOs exceeds the available capacity of the PON, then the PON operator informs the MOs of his inability to satisfy their requests, either by simply rejecting them or by proposing a counter-offer of a feasible allocation. The counter offer may refer to a proportional allocation based on the initially requested resources, or on any other scheme described in the SLA between the PON operator and the MO. Hence, the MOs will either request for less resources or they will accept the counter offer. As a result, the MOs will reduce their demand from the PON. This reduction may correspond to different scenarios, such as the MOs offering a degraded QoS to their customers, or enabling the handover process in order to transfer a portion of the subscribers to neighbouring BSs, thus reducing the aggregated demand in the cell.

On the other hand, if the aggregated demand of the MOs is lower than the available resources of the PON, then both requests can be satisfied. However, as the MOs are informed for the availability of more resources they request for a greater proportion in order to provide an enhanced QoS to their customers and subsequently to increase their revenues. This corresponds to the scenario where the extra available resources provided to the BSs are consumed by the end-users who exploit the high QoS provisioning by requesting more services (e.g. for ftp services or video streaming). Consequently, both MOs adapt their behaviour according to the available capacity of the passive optical backhaul network without any further coordination between them, settling thus, the system to an equilibrium point.

After the equilibrium point has been achieved and the appropriate resources have been assigned to the BSs, the end-users demand can be successfully satisfied either by the PON resources or by other means as described above. However, in order to efficiently exploit the backhaul resources, after a short time, the leasing period ends, and a new negotiation phase of the resources is initiated. The appropriate leasing periods can be determined by the periodicities in the daily traffic demand pattern of the BSs, due to the habitual behaviour of the end-users [15], and should be included in the SLA between the MO and the PON operator.

IV. Time Delay model

Through the analysis of the previous section, the game under consideration was proven to be asymptotically stable under the replicator dynamics. However, the replicator dynamics model is based on the assumption that the interactions among the players have immediate results to the game. A more realistic case would assume that the results of the interactions may affect the players with a certain delay. This delay may correspond either to the different intervals that the MOs update their

[6]The resources are requested in discrete steps as dictated in [25].

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knowledge about the network status or to software limitations of their infrastructure [27]. Assuming that the payoffs acquired at time t will impact the rates of growth τ_1 time later for MO$_A$ and τ_2 time later for MO$_A$ then the payoff of strategy i for MO$_A$ is the payoff τ_1 time units ago, whereas for MO$_B$ it is the one τ_2 units ago. The system is then formulated as seen in Eq. (4).

The first term inside the brackets in Eq. (4) is the delayed payoff of the pure strategy i and j for the MO$_A$ and MO$_B$, respectively, while the second term is the corresponding average payoff. The validity of the following theorem can be proven.

Theorem 4.1: Under the delays of τ_1 and τ_2, the MOs' requests will converge to the available resources of the PON.

Proof: The proof is included in Appendix C. ■

Theorem 4.1 is important, since it reveals that the game will be led to an equilibrium state whereby the MOs' aggregated demand converges to the available resources of the PON, even under the effect of time delays.

V. Simulation Results

A. Simple Game with two BSs

The simulation results provided in this subsection aim at the validation of the proposed system design. To accomplish this, a set of payoff functions that are consistent with the description of Section II is selected, and the players' profiles are also composed. In particular, let us consider that the gain $S_z^{(i)}$, $z \in \{A, B\}$, for the MOs is expressed as an exponential function of the requested class $\mathcal{C}_i$. In other words,

$$S_z^{(i)} = \epsilon_z \cdot e^{(i)\omega_z} \tag{5}$$

where i refers to the requested class $\mathcal{C}_i$ and $\epsilon_z, \omega_z > 0$ constitute parameters that depend on the MOs' profile. Additionally, the resources' cost is considered to be exponential as well[7], with regards to the requested class $\mathcal{C}_i$, thus

$$C^{(i)} = \mu \cdot e^{(i)\nu} \tag{6}$$

where i refers to the requested class $\mathcal{C}_i$ and $\mu, \nu > 0$ constitute parameters that depend on the PON operator's pricing policy. The selected function forms are similar to the ones used in [28] for modelling and studying a network provider's satisfaction in the case of the radio access technology selection optimisation problem in heterogeneous wireless environments. Other payoff functions such as the linear form function or the logarithmic form function can also be applied provided that they comply with the constraints described in the previous sections. The selection of such functions depends on the profile of the MOs and the end-users that they serve, as well as the charging policy of the PON operator. For instance, an exponential form function gives more weight to higher classes of IR and may refer to big MOs who want to maintain their reputation by offering higher data rates, while a logarithmic function gives weight to lower classes and may refer to a smaller MO with fewer customers to serve, who provides lower data rates.

[7]The exponential functions constitute an ideal selection to depict the steep increased of the payoffs of the MOs for higher classes $\mathcal{C}_i$, along with the increase in the cost of the respective resources, and they have been widely used in the literature [28].

Consider an indicative case with $N = 7$ classes, with parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.45$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$. The above parameter values formulate different profiles for the MOs and may correspond to different charging policies for their offered services, as explained in Section II. Specifically, it is assumed that MO$_A$ experiences a higher payoff than the MO$_B$ for the same amount of resources. In Figure 2 and in Figure 3, the evolution of the strategies of the BSs of MO$_A$ and MO$_B$ are depicted, respectively. One may notice the fast convergence of the aggregated resources requested by the two MOs to the available resources of the PON. In particular, the convergence speed increases when a certain strategy experiences a payoff which is greater than the payoffs of all the other strategies. Specifically, for the selected functions, the convergence speed increases when the gain $S_z^{(i)}$, $z \in \{A, B\}$ increases, i.e. when ϵ_z or ω_z increase. Apart from that, the same result can be achieved by lowering the cost of the resources $C^{(i)}$, i.e. when μ or ν decrease.

While this convergence is assured, the specific classes in which the BSs' requests are going to converge depend on the set of initial conditions, i.e. their estimated subscribers' demand, as well as the equilibrium points' basins of attraction. In particular, the system will end up to the equilibrium state that lies in the same basin of attraction as the initial states. The size of these basins depends on the players' payoffs, as defined by the parameter values. The reader that is interested in methods for computing the basins of attraction, which due to complexity is beyond the scope of this paper, may refer to [29]-[31] for more details.

In Figure 4 and in Figure 5 the impact of the time delay on the game's convergence is depicted. Specifically, it can be seen that time delay negatively affects the speed of convergence. It is widely known that time delays affect the convergence of dynamical systems, resulting to oscillations or even change the stability of the system for high values of delay, as shown in [27]. Nevertheless, Theorem 4.1 ensures that, even for high time delays τ_1 and τ_2, system convergence to an equilibrium will be achieved.

B. Game with three BSs

In the previous subsection we used a simple example to present the validity of the proposed scheme. In order to elaborate more on the model let us extend the previous scenario and consider that another BS belonging to MO$_A$ is connected to the same ONU. It is noted that the game is played at the backhaul optical network, and as a result, the requests of each MO are the aggregated requests of the BSs connected to the ONU. Thus, assume that the first BS of MO$_A$ requests for class $\mathcal{C}_i$ while the second BS of MO$_A$ requests for class $\mathcal{C}_j$, then the aggregated demand of the MO$_A$ would belong to class $\mathcal{C}_{i+j}$ yielding a payoff function described by

$$\mathcal{U}_A^{(i+j)} = S_A^{(i+j)} - C^{(i+j)} \tag{7}$$

Let us also assume for consistency that the payoff functions remain the same as in Eq. (5) and Eq. (6). Because the committed resources of MO$_A$ will be shared to more than one BSs in the current scenario, it is expected that the

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$$\dot{x_i}(t) = x_i(t) \cdot \left[\sum_{j=1}^{N} A_{i,j} y_j(t-\tau_1) - \sum_{k=1}^{N} \sum_{m=1}^{N} x_k(t) A_{k,m} y_m(t-\tau_1) \right] \tag{4a}$$

$$\dot{y_j}(t) = y_j(t) \cdot \left[\sum_{i=1}^{N} B_{i,j} x_i(t-\tau_2) - \sum_{k=1}^{N} \sum_{m=1}^{N} y_k(t) B_{m,k} x_m(t-\tau_2) \right] \tag{4b}$$

$$\text{for } i, j = 1, \ldots, N$$

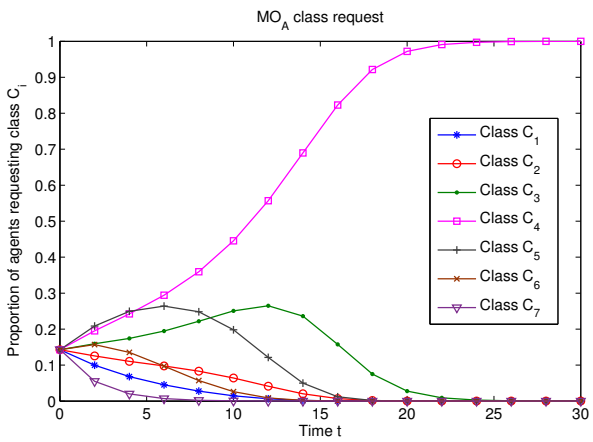

Fig. 2. Evolution of the requested classes of MO_A for parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.45$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$.

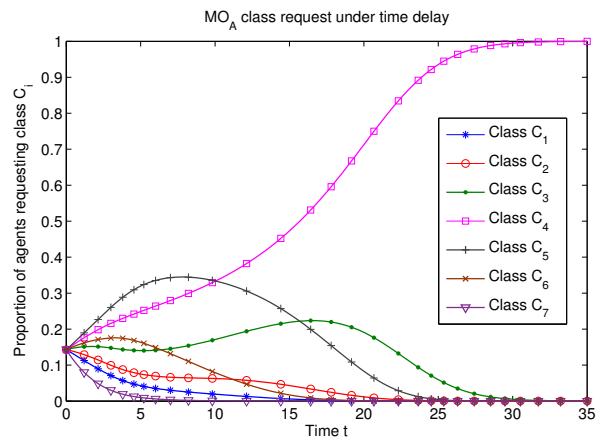

Fig. 4. Evolution of the requested classes of MO_A under time delay $\tau_1 = 2$ and $\tau_2 = 3$, for parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.45$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$.

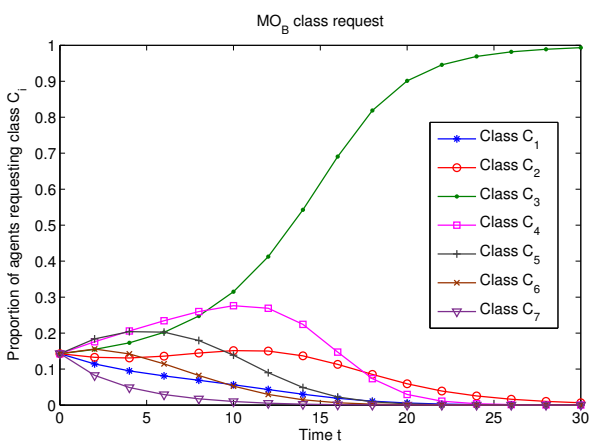

Fig. 3. Evolution of the requested classes of MO_B for parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.45$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$.

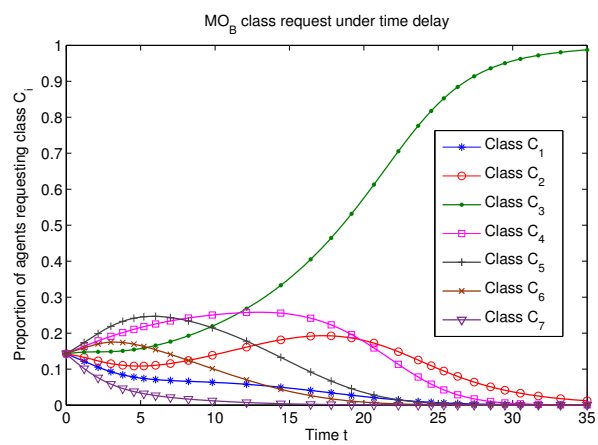

Fig. 5. Evolution of the requested classes of MO_B under time delay $\tau_1 = 2$ and $\tau_2 = 3$, for parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.45$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$.

MO will have different perspective concerning the resources. Specifically, MO_A would experience a higher payoff when both BSs can be successfully served. This can be depicted by using $\epsilon_A = 0.5$ and $\omega_A = 0.55$ in Eq. (5).

Considering the same case with $N = 7$ classes as in subsection V-A and parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.55$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$, the results of the evolution of strategies for MO_A and MO_B are depicted in Figure 6 and in Figure 7, respectively. It can be easily seen that the game is settled to a new equilibrium point where MO_A requests a higher class which corresponds to a higher IR in order to serve the needs of both BSs connected to the ONU. The same results are also obtained for the case of delayed dynamics, which are omitted due to space constraints.

VI. Conclusion and Future Research

The current paper focused on the backhaul resource allocation problem in a converged wireless - optical network architecture. In particular, the case in which two competing

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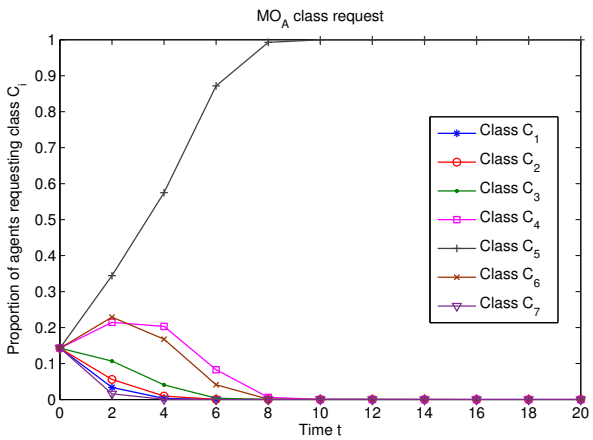

Fig. 6. Evolution of the requested classes of MO_A serving two BSs, for parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.55$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$.

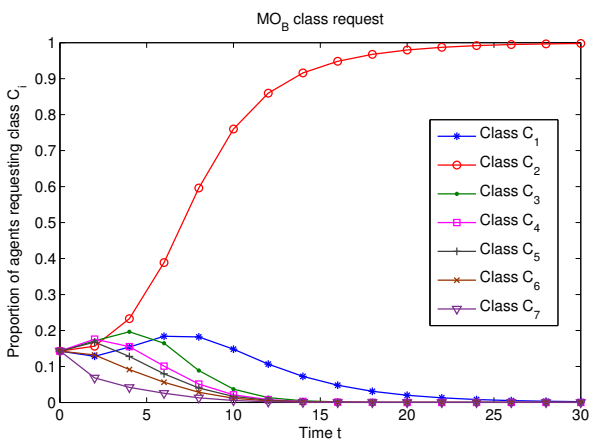

Fig. 7. Evolution of the requested classes of MO_B serving two BSs, for parameter values $\epsilon_A = 0.5$, $\epsilon_B = 0.4$, $\omega_A = 0.55$, $\omega_B = 0.45$, $\mu = 0.3$, $\nu = 0.4$.

BSs that belong to different MOs and share a common optical backhaul network infrastructure was investigated. The proposed approach made use of evolutionary game theory for modelling the interactions among the BSs and the PON operator. The paper proved, through the replicator dynamics methodology, that the requested demand by the BSs will converge to the available resources of the PON. In this way, the BSs will request the appropriate IR in order to provide a guaranteed QoS to their subscribers, considering also the limitations of the backhaul optical network. Finally, the study and results assured that the proposed system design maintains its asymptotic stability nature even in the presence of time delay.

For future research, the authors plan to investigate on the problem of end-to-end QoS taking into consideration the limitations of each underlying network type (i.e. access network and backhaul network) and providing a unified model that describes the interactions among the different network elements and the users. Apart from that, another interesting problem is the investigation on the system dynamics with more than two MOs. Specifically, the MOs can either compete among themselves for the resources of the backhaul network or form coalitions. This problem extends the current approach and the authors plan to investigate on it in a future work.

Appendix A
Proof of Theorem 3.1

It can be noticed that the pairs $(\mathcal{C}_k, \mathcal{C}_{N-k})$ for $k \in [1, \ldots, N-1]$ are pure strategy Nash Equilibria (NE), and there also exists one mixed strategy NE. Based on [23] the mixed-strategy NE cannot be an asymptotically stable equilibrium under the replicator dynamics, and, as a consequence, it is not considered further. Referring back the former equilibrium points, by combining linearisation techniques and simple block matrices algebra [32], it can be easily found that the system's eigenvalues for the k-th equilibrium point $(\mathcal{C}_k, \mathcal{C}_{N-k})$, $k \in [1, \ldots, N-1]$, are as follows

$$\lambda_i^{(k)} = \begin{cases} (A_{i,N-k} - A_{k,N-k}) & \text{if } i \neq k \\ (A_{N,N-k} - A_{k,N-k}) & \text{if } i = k \end{cases} \tag{8a}$$

$$\lambda_{N-1+i}^{(k)} = \begin{cases} (B_{k,i} - B_{k,N-k}) & \text{if } i \neq k \\ (B_{k,N} - B_{k,N-k}) & \text{if } i = k \end{cases} \tag{8b}$$

for $i = 1, \ldots, N-1$. Thus, in conjunction with the assumptions of Section II, all the eigenvalues have negative real part, which means that the $N-1$ pure strategy NE, $(\mathcal{C}_k, \mathcal{C}_{N-k})$ for $k \in [1, \ldots, N-1]$, are asymptotically stable and, consequently, the MOs' aggregated demand will converge to the available resources of the PON.

Appendix B
Proof of Theorem 3.2

Based on the proof described above, there are equilibrium points $(\mathcal{C}_k, \mathcal{C}_{N-k})$ for $k \in [1, \ldots, N-1]$, with $\mathcal{C}_k > \mathcal{C}_{thr}^A$ or $\mathcal{C}_{N-k} > \mathcal{C}_{thr}^B$, for which the eigenvalues described by Eq. (8) become positive, and the point becomes unstable. As a result, the set of equilibrium points reduces to a subset $(\mathcal{C}_k, \mathcal{C}_{N-k})$, where the classes are below the given thresholds, i.e. $\mathcal{C}_k < \mathcal{C}_{thr}^A$ and $\mathcal{C}_{N-k} < \mathcal{C}_{thr}^B$.

Appendix C
Proof of Theorem 4.1

The system experiences the same equilibrium points as in Appendix A. By means of linearisation techniques, the characteristic polynomial [33] calculated at the k-th equilibrium point is the following

$$| \mathbf{J}_0^k + e^{-\lambda\tau_1}\mathbf{J}_{\tau_1}^k + e^{-\lambda\tau_2}\mathbf{J}_{\tau_2}^k - \lambda\mathbf{I} | = 0 \tag{9}$$

where $\mathbf{J}_0^k$ is the Jacobian of the system calculated with regards to $(\mathbf{x}_k(t), \mathbf{y}_k(t))$, $\mathbf{J}_{\tau_1}^k$ is the Jacobian calculated with regards to $(\mathbf{x}_k(t-\tau_1), \mathbf{y}_k(t-\tau_1))$ and $\mathbf{J}_{\tau_2}^k$ is the Jacobian calculated with regards to $(\mathbf{x}_k(t-\tau_2), \mathbf{y}_k(t-\tau_2))$.

Similarly to the process followed in Appendix A, it can be found that $\mathbf{J}_{\tau_1}^k = \mathbf{0}$ and $\mathbf{J}_{\tau_2}^k = \mathbf{0}$ at the equilibrium points. In consequence, the system's stability analysis is the same as in Appendix A.

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