Computing Blocking Probabilities in Survivable Elastic Optical OFDM Networks

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***Abstract*—We present an approximate analytical method to compute the performance measurements in optical orthogonal frequency-division multiplexing (OFDM) networks with survivability. Our approach divides the network into layers and the equivalent path technique is used to model each path as an equivalent single link system. The analysis of the performance measurements for an equivalent single link system is based on a superposition concept using the Kaufman/Delbrouck recursion model. We assume static routing with a first-fit spectrum allocation. Simulation results that indicate the accuracy of our method are presented.**

***Index Terms*—Performance measurements; OFDM-based optical networks; Routing and Spectrum allocation (RSA)**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) is currently used in most new and emerging networks to transport several tens of gigabits and terabits of data per second in a cost-effective and scalable manner [1]. An optical network architecture known as the spectrum-sliced elastic optical path (SLICE) network was proposed to support various client services ranging from subwavelength to superwavelength services [2]. The OFDM-based elastic optical network divides a communication channel into a number of equally spaced frequency bands and a subcarrier carrying a portion of the user information is transmitted in each band. Each of these subcarriers is orthogonal (i.e., independent of the other subcarriers). OFDM optical networks allocate an appropriate number of these subcarriers to accommodate both the subwavelength and the superwavelength traffic [3]. Thus, a connection requiring a capacity larger than one OFDM subcarrier must be allocated a number of contiguous subcarriers (slots) in a process known as spectrum allocation (Fig. 1). Hence, the problem of serving a connection that requires x subcarriers translates into a problem of finding a route with a starting subcarrier frequency that can then accommodate x contiguous subcarriers (in addition to the guard bands). Note that guard bands are not needed between the contiguous channels assigned to the same call.

Survivability is the ability to provide service in the presence of a single failure, which could be due to fiber cuts, failure of an active component inside network equipment, or node failure [4]. Such failures lead to large losses in data and revenue [5]. A network can serve two types of calls: protected and unprotected. Protected calls are designed to overcome a single network failure, most commonly by assigning a disjoint backup lightpath (*path protection*) for each working lightpath. In general, path protection is more capacity-efficient than link protection [6]. The backup path can be either a dedicated backup or a shared backup. In dedicated protection, each working path is assigned its own dedicated lightpath to which it can switch in case of a failure. In shared protection, a backup wavelength can be shared on some links as long as their protected segments (links, subpaths, and paths) are mutually diverse. For more details about survivability we refer the reader to our work in [7].

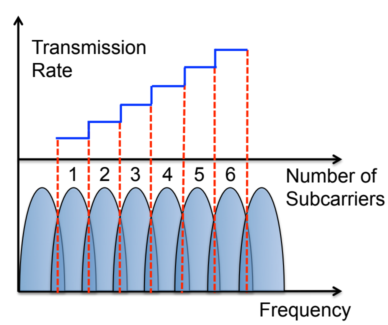


Fig. 1. In OFDM-based elastic optical networks, the data rate/bandwidth can be adjusted based on the number of subcarriers. A connection requiring a capacity larger than one OFDM subcarrier must be allocated a number of contiguous subcarriers in a process known as the spectrum allocation [1].

Some of the call requests are blocked (denied), owing to the unavailability of resources; i.e., none of the groups of the contiguous subcarriers are available. Moreover, the network performance should be estimated based on the expected traffic demands. The challenge of evaluating the network performance has been investigated in several studies [8]. The purpose of this paper is to compute the network performance in a survivable OFDM optical network and to the best of our knowledge, this is the first study to address this issue. It is an extension to the algorithm introduced in [8], to compute the network performance of an OFDM optical network without protection. Dedicated path protection is used for protected traffic; both working and backup paths are predetermined and fixed at the design stage.

The remainder of this paper is organized as follows: first, we introduce the network model, traffic assumptions, and performance measurements. Then, we review the three main concepts behind our proposed algorithm. Next, we describe our proposed calculations for the optical OFDM network. Section V presents illustrative examples and the numerical results. We present our conclusions in the last section.

II. Network Model

There are I different call types, each with different subcarrier requirements T_i, $i \in I$. For example, the Telefonica National Network has links that are capable of transporting a capacity of 4 THz and the bit rate for the connection requests is uniformly selected from rates of 40, 100, 160, 400, and 600 Gb/s [9]. The call arrival process is assumed to be Poissonian and the call holding time is exponentially distributed with a mean value equal to one unit of time (e.g., 2 h) [10]. In addition, we assume that the blocked calls are lost and do not attempt to re-enter the system. We also assume that the call connection and disconnection times are short in comparison with the call holding time; hence, these times can be neglected. As mentioned above, serving a connection that requires T_i subcarriers translates to finding a route with a starting subcarrier frequency, f, after which it can use T_i contiguous subcarriers (in addition to the guard bands) [1]. Further, a first-fit spectrum allocation in which the starting subcarrier frequency, f, is searched in a fixed order, is used. Without loss of generality, let the first-fit order for a type, i, start with the number 1 (*called stage 1*). Further, if the traffic offered to the starting subcarrier frequency, 1, is blocked, then the traffic will try the starting subcarrier frequency, $T_i + 1$ (i.e., stage 2). If that too is unavailable, then the traffic will try the starting subcarrier frequency $2T_i + 1$, i.e., stage 3 and so on until the call is either accepted or rejected. This assumption introduces a "step" behavior in which a number of contiguous subcarriers have the same utilization. Nevertheless, it is proven to provide the same or better performance than the traditional first-fit method [11]. Thus, the first-fit order for type i is $1, T_i + 1, 2T_i + 1, \ldots$ etc. The following notations and assumptions will be used throughout this paper:

- C is the link capacity, that is, the number of available subcarriers in a link and it is uniform across all the links in the network.
- I is the number of different call types, with each having different subcarrier requirements, T_i, $i \in I$. Without loss of generality, we assume $T_i < T_{i+1}$.
- C'/C is the number of layers, where each layer will have a capacity of C'.
- $\mu = 1$ is the mean of the (exponentially distributed) request holding time.
- $\lambda_i(s,d)$ and $\ddot{\lambda}_i(s,d)$ are the protected and unprotected type $i \in I$ Poisson arrival rate from the source node, s, to the destination node, d.
- Λ_i is defined as the ratio of the type i arrival rate to the total arrival rate of all the calls i.e., $\Lambda_i = [\sum_{\forall s,d} \lambda_i(s,d) + \ddot{\lambda}_i(s,d)]/[\sum_{\forall s,d,i} \lambda_i(s,d) + \ddot{\lambda}_i(s,d)]$.
- w_i is the revenue generated by carrying (*accepting*) a call type, i. We assume that the value of the request equals the request capacity, i.e., $w_i = T_i$ \$.
- $A_i(s,d)$ and $\ddot{A}_i(s,d)$ are, for call type i, the protected and unprotected single link equivalent offered loads of the source/destination pairs s, d.
- $R(s,d)$ and $\ddot{R}(s,d)$ denote the working and backup paths for the source/destination pair s, d.
- $a_{y,j}$ is the sum of all background traffic from all the source/destination pairs on the segment, $R(y,j)$.
- $P_i(y,j)$ is the blocking probability for call type i on the segment $R(y,j)$ belonging either to the working or the backup paths, i.e., $R(y,j) \in R(s,d)$ or $R(y,j) \in \ddot{R}(s,d)$, respectively.
- $B_i(s,d)$ and $\ddot{B}_i(s,d)$ are the unprotected and protected end-to-end call type i blocking probability for the source-destination pair, s, d.

The key criterion used for measuring the network performance is the overall revenue (in \$) lost because of the blocked requests [12]. The lost revenue L is the sum of all the end-to-end probabilities of the blocking type, $i \in I$, multiplied by the value of such a request, $w_i = T_i$, and the arrival rate.

$$L = \sum_{i=1}^{I} T_i \sum_{s,d} \lambda_i(s,d)\ B_i(s,d) + \ddot{\lambda}_i(s,d)\ \ddot{B}_i(s,d) \quad \$ \tag{1}$$

The total revenue is $W = \sum_{i=1}^{I} T_i \sum_{s,d} [\lambda_i(s,d) + \ddot{\lambda}_i(s,d)]$ \$ and the normalized lost revenue is $\hat{L} = L/W$. The targeted overall blocking probability (Grade of service GOS) is between 1% and 10% [13].

III. Background

For completeness, we briefly review the three key concepts that will be used to derive our method.

A. Single Link Blocking Probabilities in OFDM

In this section, we review the results given in [12] to calculate the blocking probabilities in a single link in an OFDM optical network. The algorithm divides the link capacity into stages of size T_i for each call type i. In each stage, three traffic streams will be offered: the type i overflow load from the previous stage, the outer background traffic, and the inner background traffic. The inner background traffic is for calls with smaller sizes; hence, they can be carried on portions of the stage, whereas, the outer background traffic, for calls with larger sizes, occupies all the subcarriers of the stage. If the traffic offered to a stage is blocked, then the traffic will try the next stage without any overlapping subcarriers between the stages. The Kaufman [14] (resp. Delbrouck [15]) recursion formula is used to calculate the stage blocking probability by considering all the inner background traffic and ignoring the outer background traffic. The Kaufman (resp. Delbrouck) recursion formula is an excellent approximation for calculating the stage blocking probability when the stage size is equal to the call size, T_i [12]. The Kaufman recursion formula is used when the traffic is Poissonian and the Delbrouck recursion formula is used when the traffic is non-Poissonian. Next, the carried outer background traffic "superimposes" the carried outer background traffic over the stage blocking probability, calculated by using the inner background traffic. The carried traffic is superimposed over the blocking probabilities if the traffic is aligned on multiple stages; otherwise, each lower stage is extended until it is aligned with the upper stage. Finally, the following notation is used to present the output of the single link blocking probabilities calculation [12];

$$B_1, B_2, \ldots, B_I \leftarrow f_B(I, T, \lambda, C) \tag{2}$$

B. Path Blocking Probability

In this section, we review the results given in [16] to calculate the blocking probabilities in a single layer

1) Segment Background Traffic: Figure 2 shows a path $R(1,k)$ with background traffic generated from other source/destination (s,d) pairs. Assume segment $R(l,m)$ is a subset of path $R(1,k)$, i.e., $R(l,m) \subseteq R(1,k)$ and segment $R_{l,m}$ is also a subset of s,d path, i.e., $R(l,m) \subseteq R(s,d)$. Further, traffic passing through segment $R(l,m)$ does not pass through any other segment belonging to path $R_{(1,k)}$. Using the reduced blocking path model to account for the traffic thinning in other links [18] [17], type i background traffic $a_i(l,m)$ generated from protected and unprotected calls, is conditioned on the availability of other segments belonging to the path $R(s,d)$ or $\ddot{R}(s,d)$ and not part of segment $R_{l,m}$. Thus, the mean background traffic $a_i(l,m)$ passing through segment (l,m) can be written as

$$a_i(l,m) = \frac{\displaystyle\sum_{s,d} \lambda_i(s,d)(1 - B_i(s,d)) + \ddot{\lambda}_i(s,d)(1 - \ddot{B}_i(s,d))}{[1 - P_i(l,m)]} \tag{3}$$

where, $P_i(l,m)$ is the type i blocking probability of segment $R(l,m)$.

2) Path Blocking Probability : In this work, we assume that mean background traffic $a_i(l,m)$ in Eq. 3 are independent (i.e., the link independent assumption). This assumption has been used to calculate the blocking probability in circuit-switched and WDM networks [17] [19]. We also ignore the fact that some background traffic may enter and exist on different non-consecutive segments in the paths (see more details in [7]). This simplification is to speed up the computation and will be taken into account in future research. From the work in [16] and [7], the type i blocking probability of path $R(1,k)$ (Fig. 2) can be calculated as:

$$P_i(1,k) = 1 - \frac{1}{G_i(1,k)} \tag{4}$$

where $G_i(1,k)$ is the normalization constant for type i on the path $R(1,k)$ given by the following recursive formula [16] as:

$$G_i(1,k) = G_i(1,k-1) + \sum_{l=1}^{k-1} G_i(1,l) a_i(l,k) \tag{5}$$

where $G_i(1,1) = 1$.

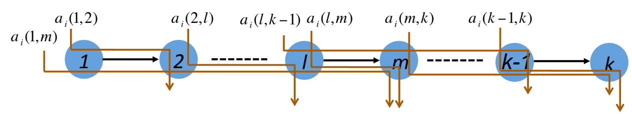


Fig. 2. A $k-1$ hop path $R(1,k)$ showing type i background traffic $a_i(l,m)$ for each segment $R_{l,m} \in R(1,k)$.

Finally, the blocking probability $\ddot{B}_i(s,d)$ of protected call type i from source s to destination d with two disjoint working and backup paths, $R(s,d)$ and $\ddot{R}(s,d)$, can be calculated as

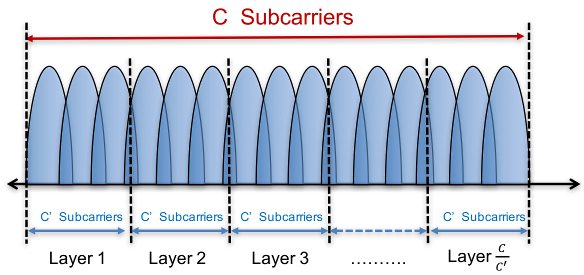
$$\ddot{B}_i(s,d) = P_i(s,d) \times \ddot{P}_i(s,d) \tag{6}$$

Obviously, $B_i(s,d) = P_i(s,d)$.

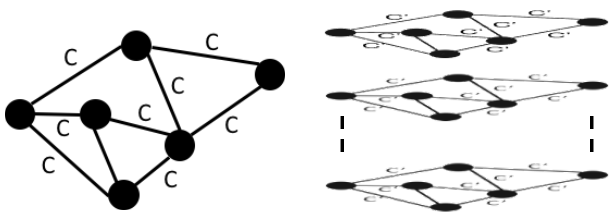
C. Equivalent path model

The equivalent path model was first presented by Dziong to solve for the circuit-switched network blocking probabilities [20]. In [21], the equivalent path model was used to solve for the WDM network blocking probabilities. The model consists of replacing every path in the network by an equivalent single link system such that the blocking traffic in this system will approximate the blocking on the path [17]. Thus, from Eq. 4, the type i equivalent single link offered load $A_i(1,k)$ is given as

$$A_i(1,k) = G_i(1,k) - 1. \tag{7}$$



(a) A single link view.



(b) A network view.

Fig. 3. The proposed algorithm divides the network with a link capacity, C, into C/C' layers, each with a capacity of C'. A layer is a subgroup of C' subcarriers.

IV. Proposed Algorithm

In this section, we present our proposed algorithm to solve for the normalized lost revenue in an optical OFDM network. First, we divide the network with C link capacities into C/C' network with C' link capacities as shown in Fig. 3 [8] [12]. A preferred choice for $C' = T_I$ i.e., the size of the largest call request. Dividing the network into more layers can achieve higher accuracy when calls with lower bandwidth requirement have higher arrival rates. Clearly, more layers means more computational complexity. Once the blocking probabilities are calculated in a layer, we calculate the overflow traffic to the next layer as the product of the blocking probability and the mean traffic offered to the layer.

Using the mean to calculate the overflow traffic will underestimate the blocking probabilities [17]. However, the link-independent assumption in section III-B2 overestimates the call blocking probabilities, particularly, when the nodal degree of the network is small [16]. Thus, the underestimating effect of using the mean overflow is balanced by the overestimating effect of the link-independent assumption. This is similar to calculating the blocking probabilities with the first-fit in a WDM network using the link-independent assumption and mean overflow traffic [19].

In each layer, we apply the equivalent path model discussed in III-C to solve for the blocking probabilities. The traffic offered to the equivalent single link system are calculated using the technique discussed in section III-B and the equivalent single link blocking probabilities are calculated using the technique discussed in section III-A. The pseudocode is shown in Algorithm 1.

Algorithm 1 Proposed algorithm.

Require: Number of layers C/C' and $\epsilon << 1$.

Assume initial values for end-to-end blocking probabilities and diff $>> \epsilon$.
Do
  for $Layer = 1$ **to** C/C'
    for $\forall s, d$ **and** $\forall i \in I$
      Calculate $a_i(l, m)$ to each segment l, m (Eq. 3).
      Calculate $A_i(s, d)$ and $\ddot{A}_i(s, d)$ (Eqs. 5 and 7).
    end for
    Calculate new $B_i(s, d)$ and $\ddot{B}_i(s, d)$ (Eq. 2).
    Calculate the overflow traffic to the next layer $\lambda_i(s, d) \leftarrow \lambda_i(s, d) \times B_i(s, d)$ and $\ddot{\lambda}_i(s, d) \leftarrow \ddot{\lambda}_i(s, d) \times \ddot{B}_i(s, d)$.
  end for
  diff = the difference between the new end-to-end blocking probabilities and the old ones.
while (diff $> \epsilon$)
Use Eq. 1 to calculate $\hat{L}$.

V. Numerical Results

To validate our analytical method, we use a discrete event simulation that is performed and repeated 20 times, wherein each run begins with a different random seed. The averaged result is obtained with a 95% confidence and a confidence interval of less than 1% of the average value.

First, we introduce an illustrative example for a simple network shown in Fig. 4 with $C = 4, I = 3, T = \{1, 2, 4\}$ and $\lambda_i(1, 2) = \lambda_i(1, 3) = \lambda_i(2, 4) = \lambda_i(3, 4) = \ddot{\lambda}_i(1, 4) = 0.5 \ \forall \ i \in I$. Only calls from source 1 to destination 4 are protected, where the working path is through node 2 and the backup path is through node 3. Paths $R(1, 2)$, $R(1, 3)$, $R(2, 3)$, and $R(2, 4)$ have similar equivalent single link and blocking probabilities (*the network is symmetrical*). For instance, the offered load for path $R(1, 2)$ is calculated as

$$A_i(1, 2) = \lambda_i(1, 2) + \frac{\ddot{\lambda}_i(1, 4)[1 - \ddot{B}_i(1, 4)]}{1 - P_i(1, 2)}. \qquad (8)$$

The working path $R(1, 4)$ and backup path $\ddot{R}(1, 4)$ equivalent single link are similar (i.e., $A_i(1, 4) = \ddot{A}_i(1, 4)$ since the network is symmetrical) and are calculated as;

$$A_i(1, 4) = \lambda_i(1, 2) + \lambda_i(2, 4) + \lambda_i(1, 2) * \lambda_i(2, 4) + \frac{\ddot{\lambda}_i(1, 4)[1 - \ddot{B}_i(1, 4)]}{1 - P_i(1, 4)} \qquad (9)$$

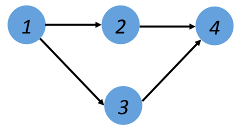
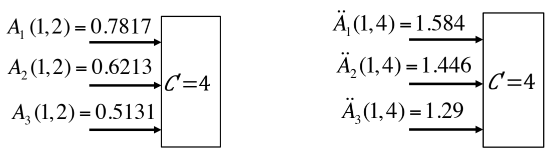


Fig. 4. An illustrative example of a network with four links and $C = 4, I = 3, T = \{1, 2, 4\}$ and $\lambda_i(1,2) = \lambda_i(1,3) = \lambda_i(1,4) = \lambda_i(2,4) = \lambda_i(3,4) = 0.5 \; \forall \; i \in I$. Calls from source 1 to destination 4 are protected, where the working path is through node 2 and the backup path is through node 3

We iteratively solve for the equivalent single link offered load (e.g., Eq. 8) and the path blocking probabilities using Eq. 2 (e.g., $f_B(I = 3, T = \{1, 2, 4\}, A_i(1, 2), 4)$). The results are shown in Fig. 5. The simulation and calculation blocking probabilities are given in Table I. The total revenue is $W = 17.5$ and the normalized lost revenue are $\hat{L} = 63.23\%$ and $\hat{L} = 63.26\%$ from simulation and calculation, respectively.



(a) $R(1, 2), R(1, 3), R(2, 4)$ and $R(3, 4)$

(b) $R(1, 4)$ and $\ddot{R}(1, 4)$

Fig. 5. The equivalent single link for each path in the network shown in Fig. 4 with $C = 4$.

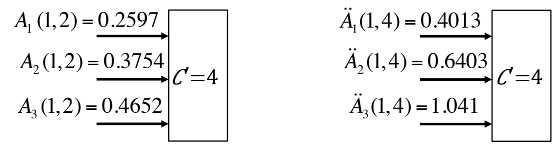
TABLE I

THE OVERALL BLOCKING PROBABILITIES FOR FOR THE ILLUSTRATIVE EXAMPLE WITH $C = 4$. THE NORMALIZED LOST REVENUES ARE $\hat{L} = 63.23\%$ AND $\hat{L} = 63.26\%$ FOR SIMULATION AND THE PROPOSED ALGORITHM, RESPECTIVELY. THE PROPOSED ALGORITHM USES AN EQUIVALENT SINGLE LINK OF $C' = 4$.

	$i = 1$		$i = 2$		$i = 3$	
Path	Sim.	Prop.	Sim.	Prop.	Sim.	Prop.
1-2/1-3/2-4/3-4	0.208	0.208	0.378	0.364	0.744	0.761
1-4 (protected)	0.667	0.554	0.869	0.846	0.991	0.994
Average	0.300	0.277	0.476	0.460	0.793	0.807

Now, we increase the link capacities from the previous example to $C = 8$ subcarriers. There are two ways to solve the problem, first we divide the network into two layers, each of capacity $C' = 4$ subcarriers. Thus, the second layer will receive the overflow traffic from the first layer calculated previously. The equivalent single link for each path in the second layer, (subcarriers $5 - 8$) using Eqs. 8 and 9 are shown in Fig. 6. The blocking probabilities from simulation and calculation are given in Table II. The normalized lost revenue are $\hat{L} = 38.98\%$ and $\hat{L} = 37.28\%$ from simulation and calculation, respectively.

An alternative way to solve for the blocking probabilities is to solve it as a one layer system, i.e., $C' = C = 8$ subcarriers. The blocking probabilities from simulation and calculation are given in Table III. The normalized lost revenue is $\hat{L} = 38.53\%$ from calculation. The calculations based on two layers ($C' = 4$) tend to underestimate the blocking probabilities and the calculations based on one layer ($C' = 8$) tend to overestimate the blocking probabilities.



(a) $R(1, 2), R(1, 3), R(2, 4)$ and $R(3, 4)$

(b) $R(1, 4)$ and $\ddot{R}(1, 4)$

Fig. 6. The equivalent single link for each path in the second layer, (subcarriers $5 - 8$) for the network shown in Fig. 4 with $C = 8$.

TABLE II

THE OVERALL BLOCKING PROBABILITIES FOR THE ILLUSTRATIVE EXAMPLE WITH $C = 8$. THE NORMALIZED LOST REVENUES ARE $\hat{L} = 38.98\%$ AND $\hat{L} = 37.28\%$ FOR SIMULATION AND THE PROPOSED ALGORITHM, RESPECTIVELY. THE PROPOSED ALGORITHM USES AN EQUIVALENT SINGLE LINK OF $C' = 4$.

	$i = 1$		$i = 2$		$i = 3$	
Path	Sim.	Prop.	Sim.	Prop.	Sim.	Prop.
1-2/1-3/2-4/3-4	0.046	0.047	0.144	0.101	0.453	0.433
1-4 (protected)	0.301	0.312	0.588	0.567	0.896	0.921
Average	0.097	0.099	0.233	0.194	0.542	0.530

TABLE III

THE OVERALL BLOCKING PROBABILITIES FOR THE ILLUSTRATIVE EXAMPLE WITH $C = 8$. THE NORMALIZED LOST REVENUES ARE $\hat{L} = 38.98\%$ AND $\hat{L} = 38.53\%$ FOR SIMULATION AND THE PROPOSED ALGORITHM, RESPECTIVELY. THE PROPOSED ALGORITHM USES AN EQUIVALENT SINGLE LINK OF $C' = 8$.

	$i = 1$		$i = 2$		$i = 3$	
Path	Sim.	Prop.	Sim.	Prop.	Sim.	Prop.
1-2/1-3/2-4/3-4	0.046	0.077	0.144	0.173	0.453	0.412
1-4 (protected)	0.301	0.312	0.588	0.583	0.896	0.930
Average	0.097	0.124	0.233	0.255	0.542	0.516

Our next testing vehicle is a topology similar to the 14-node National Science Foundation network (NSFNET) shown in Fig. 7. The traffic is uniformly distributed between 182 source–destination pairs. The working path is fixed and selected from the set of shortest paths computed by Dijkstra's algorithm. In the same way, the protected path is fixed and selected from the set of shortest paths computed by Dijkstra's algorithm after removing all the edges belonging to the working path.

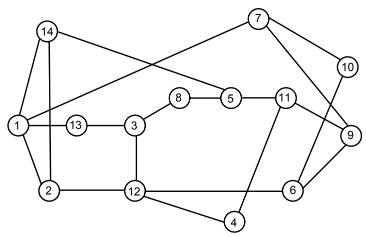


Fig. 7. A 14-node mesh OFDM optical network with link capacities $C = 32$ subcarriers. The network is offered four types of calls ($I = 4$) with subcarrier requirements of 1, 2, 4 and 8 subcarriers.

We assume the same traffic characteristics as those in [22], i.e., $C = 32$, $I = 4$, and $T = \{1, 2, 4, 8\}$. There are three different call type distributions: uniform, skew-left, and skew-right. In the uniform call type distribution, the arrival rate for all call types is uniform i.e., $\Lambda_i = 1/4$. In the skewed-to-the-left call type distribution, calls with a lower bandwidth requirement have higher arrival rates (i.e., $\Lambda_1 = 0.4$, $\Lambda_2 = 0.3$, $\Lambda_3 = 0.2$, and $\Lambda_4 = 0.1$); whereas, calls with a higher bandwidth requirement have a higher arrival rates in the skewed-to-the-right call type distribution (i.e., $\Lambda_1 = 0.1$, $\Lambda_2 = 0.2$, $\Lambda_3 = 0.3$, and $\Lambda_4 = 0.4$). These distributions were chosen not because they resemble real traffic type distributions but because of their representativeness in stochastic analysis.

The proposed algorithms will divide the network into one ($C' = 32$), two ($C' = 16$), four layers ($C' = 8$) or eight layers ($C' = 4$). In Figs. 8, 9, and 10, the normalized lost revenue, $\hat{L}$, is plotted against various protected call ratios, for uniform, skew-left and skew-right call distributions, respectively. The first curve in all the figures is obtained from simulation and the rest of the curves present our calculations based on a single layer ($C' = 32$), two layers ($C' = 16$), four layers ($C' = 8$) and eight layers ($C' = 4$). The calculations based on one layer, is close to the simulation results in the uniform and the skew-right call distributions, whereas, the calculations based on four layers, are close to the simulation results in the skew-left call distribution. Thus, for the skew-left call distribution where calls with a lower bandwidth requirement have higher arrival rates, we need smaller C' to capture the blocking probabilities of calls with a lower bandwidth requirement.

Figure 11 shows the end-to-end blocking probabilities for various calls in the mesh network with uniform call type distribution, total revenue $W = 70$ and protected calls ratio$= 0.4$. The unprotected and protected calls are numbered in ascending order of their blocking probability values which are obtained using simulations. Calls with an end-to-end blocking probability of at least 10^{-4} in the simulation are shown in the figure. We can notice that the outputs of our calculations

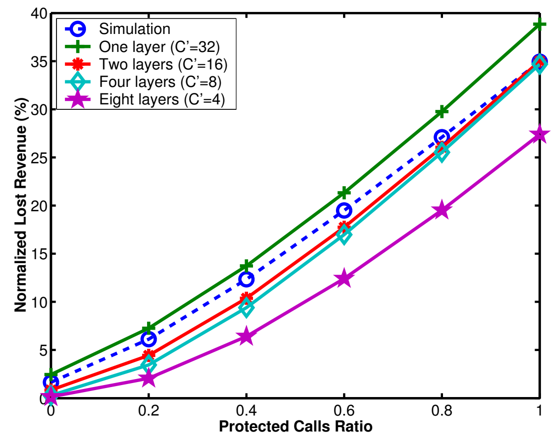


Fig. 8. The normalized lost revenue $\hat{L}$ for the network shown in Fig 7 with uniform call type distribution and total revenue $W = 70$.

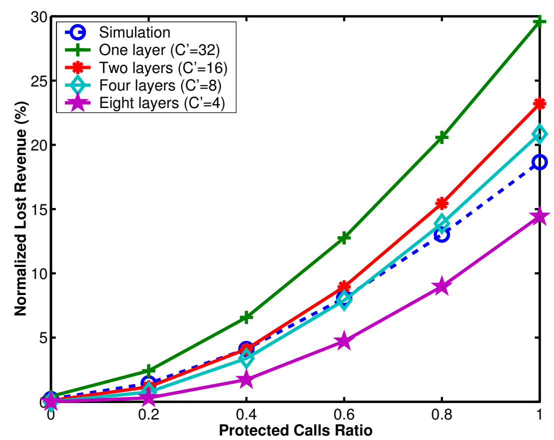


Fig. 9. The normalized lost revenue $\hat{L}$ for the network shown in Fig 7 with skew-left call type distribution and total revenue $W = 50$.

are very close to simulation results. However, when the individual blocking probability is very small, the calculation is less accurate. Nevertheless, this will not affect the model form accurately predicting the trend of overall blocking probability and the network lost revenue.

VI. Conclusion

In this paper, we expanded the equivalent path model to compute the call blocking probabilities in survivable elastic optical networks with dedicated protection, fixed routing, and first-fit spectrum allocation. The proposed model views the elastic optical network as a set of different layers where, in each layer, an equivalent single link model replaces each working and backup path. The number of layers depends on the call traffic distribution, where more layers are needed if calls with smaller bandwidth requirements have higher arrival rates. The call blocking probabilities for the equivalent single link is calculated based on the

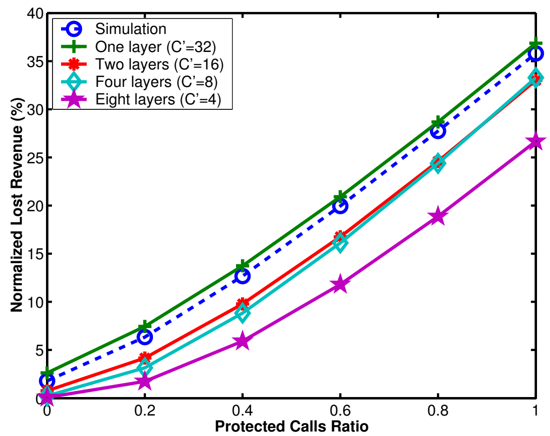


Fig. 10. The normalized lost revenue $\hat{L}$ for the network shown in Fig 7 with skew-right call type distribution and total revenue $W = 70$.

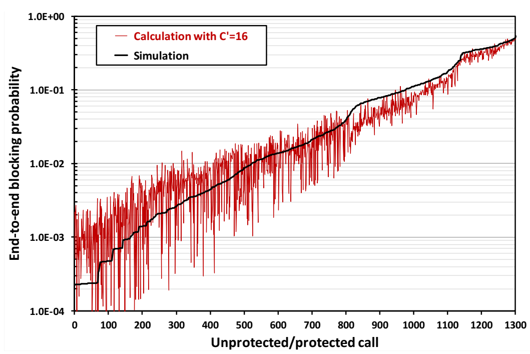


Fig. 11. The end-to-end blocking probabilities for various calls in the mesh network with uniform call type distribution, total revenue $W = 70$ and protected calls ratio$= 0.4$.

superposition concept using the Kaufman/Delbrouck recursion model. The algorithm is relatively fast and a single run will require a maximum of tens of seconds depending on the network load, traffic distribution and the number of layers on a MacBook Air (Mid 2011) with a 1.8 GHz Intel i7 processor, and 4 GB of memory.

As shown earlier, the link-independent assumption overestimates the call blocking probabilities. In future research, we plan to use the reduced load Erlang fixed-point, proposed by Kelly [18] to improve the quality of the proposed algorithm with one layer calculations. Briefly, once the unprotected and protected end-to-end blocking probabilities (i.e., $B_i(s,d)$ and $\ddot{B}_i(s,d)$), are computed, the arrival rates (i.e., $\lambda_i(s,d)$ and $\ddot{\lambda}_i(s,d)$) are modified by reducing and replacing them with the values $\lambda_i(s,d)[1-B_i(s,d)]$ and $\ddot{\lambda}_i(s,d)[1-\ddot{B}_i(s,d)]$, then new end-to-end blocking probabilities are computed. The process is repeated until a convergence criteria is satisfied [23]. Finally, the analytical model presented can be easily extended to solve for the network performance in shared path protection and multicast elastic optical networks.

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